BINOMIAL DISTRIBUTION

INFORMATION TECHNOLOGY

BAPATLA ENGINEERING COLLEGE

# The Binomial Probability Distribution

A **binomial** experiment is one that possesses the following properties:

1. The experiment consists of n repeated trials;
2. Each trial results in an outcome that may be classified as a **success** or a **failure** (hence the name, **binomial**);
3. The probability of a success, denoted by p, remains constant from trial to trial and repeated trials are independent.

* The number of successes X in n trials of a binomial experiment is called a **binomial random variable**.

## 

## Binomial Distribution

The probability distribution of the random variable X is called a **binomial distribution**.

**Formula:**

P(X)= nCx \* Px \* (1 - P)n - x

where

n = the number of trials

x = 0, 1, 2, ... n

p = the probability of success in a single trial

q = the probability of failure in a single trial

(i.e. q = 1 − p)

P(X) gives the probability of successes in n binomial trials.

## Binomial Probability

The **binomial probability** refers to the probability that a binomial experiment results in exactly *x* successes. For example, in the above table, we see that the binomial probability of getting exactly one head in two coin flips is 0.50.

Given *x*, *n*, and *P*, we can compute the binomial probability based on the binomial formula:

**Binomial Formula.** Suppose a binomial experiment consists of *n* trials and results in *x* successes. If the probability of success on an individual trial is *P*, then the binomial probability is:

b(*x*; *n, P*) = nCx \* Px \* (1 - P)n - x   
 or   
 b(*x*; *n, P*) = { n! / [ x! (n - x)! ] } \* Px \* (1 - P)n – x

## Mean and Variance of Binomial Distribution

If p is the probability of success and q is the probability of failure in a binomial trial, then the expected number of successes in n trials (i.e. the mean value of the binomial distribution) is

E(X) = μ = np

The **variance** of the binomial distribution is

V(X) = σ2 = npq

Note: in a binomial distribution, only **2** parameters, namely n and p, are needed to determine the probability.

**Example 1**  
  
 Suppose a die is tossed 5 times. What is the probability of getting exactly 2 fours?

*Solution:*

This is a binomial experiment in which the number of trials is equal to 5.

i.e; n=5

the number of successes is equal to 2. i.e; x=2.

and the probability of success on a single trial is 1/6 or about 0.167. i.e; p=0.167

Therefore, the binomial probability is:

b(*x*; *n, P*) = nCx \* Px \* (1 - P)n - x

b(2; 5, 0.167) = 5C2 \* (0.167)2 \* (0.833)3   
b(2; 5, 0.167) = 0.161

# R - Binomial Distribution

R has four in-built functions to generate binomial distribution. They are described below.

dbinom(x, size, prob)

pbinom(x, size, prob)

qbinom(p, size, prob)

rbinom(n, size, prob)

Following is the description of the parameters used −

* **x** is a vector of numbers.
* **p** is a vector of probabilities.
* **n** is number of observations.
* **size** is the number of trials.
* **prob** is the probability of success of each trial.

## dbinom()

This function gives the probability density distribution at each point.

# Create a sample of 50 numbers which are incremented by 1.

x <- seq(0,50,by = 1)

# Create the binomial distribution.

y <- dbinom(x,50,0.5)

## pbinom()

This function gives the cumulative probability of an event. It is a single value representing the probability.

# Probability of getting 26 or less heads from a 51 tosses of a coin.

x <- pbinom(26,51,0.5)

print(x)

When we execute the above code, it produces the following result −

[1] 0.610116

## qbinom()

This function takes the probability value and gives a number whose cumulative value matches the probability value.

# How many heads will have a probability of 0.25 will come out when a coin is tossed 51 times.

x <- qbinom(0.25,51,1/2)

print(x)

When we execute the above code, it produces the following result −

[1] 23

## rbinom()

This function generates required number of random values of given probability from a given sample.

# Find 8 random values from a sample of 150 with probability of 0.4.

x <- rbinom(8,150,.4)

print(x)

When we execute the above code, it produces the following result −

[1] 58 61 59 66 55 60 61 67

#### Problem

Suppose there are twelve multiple choice questions in an English class quiz. Each question has five possible answers, and only one of them is correct. Find the probability of having four or less correct answers if a student attempts to answer every question at random.

**Sol:**

**Manual calculation:**

There are twelve multiple choice questions. i.e; n=12

Each question has five possible answers, and only one of them is correct. i.e; p=1/5=0.2

Therefore q=1-p=1-0.2=0.8

The probability of having four or less correct answers if a student attempts to answers

i.e; p(x<=4)=p(x=0)+p(x=1)+p(x=2)+p(x=3)+p(x=4)

=[12C0\*(0.2)^0\*(0.8)^12]+[12C1\*(0.2)^1\*(0.8)^11]+[12C2\*(0.2)^2\*(0.8)^10]+[12C3\*(0.2)^3\*(0.8)^9]+[12C4\*(0.2)^4\*(0.8)^8]

=[0.06871]+[0.20615]+[0.28346]+[0.2362]+[0.13305]

=0.9275

***R CODE:***

#### Solution

Since only one out of five possible answers is correct, the probability of answering a question correctly by random is 1/5=0.2. We can find the probability of having exactly 4 correct answers by random attempts as follows.

> dbinom(4, size=12, prob=0.2)   
[1] 0.1329

To find the probability of having four or less correct answers by random attempts, we apply the function dbinom with x = 0,…,4.

> dbinom(0, size=12, prob=0.2) +   
+ dbinom(1, size=12, prob=0.2) +   
+ dbinom(2, size=12, prob=0.2) +   
+ dbinom(3, size=12, prob=0.2) +   
+ dbinom(4, size=12, prob=0.2)   
[1] 0.9274

Alternatively, we can use the cumulative probability function for binomial distribution pbinom.

> pbinom(4, size=12, prob=0.2)   
[1] 0.92744

* The probability of four or less questions answered correctly by random in a twelve question multiple choice quiz is 92.7%.